Tangent estimation along 3D digital curves

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Digital shapes arise naturally in several contexts e.g. image analysis, approximation, word combinatorics, tilings, cellular automata, computational geometry, biomedical imaging ...

Digital shape analysis requires a sound digital geometry which is a geometry in $\mathbb{Z}^n$
The classical problem in the digital geometry is to estimate geometrical properties of the digitalized shapes without any knowledge of the underlying continuous shape.

- length
- area
- perimeter
- convexity/concavity
- tangents
- curvature
- torsion
- ...
Many vision, image analysis and pattern recognition applications relay on the estimation of the geometry of the discrete curves.

The digital curves can be, for example, result of

- discretization
- segmentation
- skeletonization
- boundary tracking
The discrete tangent estimator evaluate tangent direction along all points of the discrete curve.
Discrete Tangent Estimator Application
In the framework of digital geometry, there exist few studies on 3D discrete curves yet while there are numerous methods performed on 2D.

- Approximation techniques in the continuous Euclidean space.
  (+) very good accuracy
  (-) require to set parameters
  (-) can be costly
  (-) poor behavior on sharp corners

- Methods which are work in discrete space directly.
  (+) good accuracy
  (+) no need to set any parameters
  (+) simple and fast
  (-) poor behavior on corrupted curves
The size of the computational window is fixed globally and is not adopted to the local curve geometry.
The size of the computational window can be adopted to the local curve geometry thanks to notion of Maximal Digital Straight Segments.
Definition

Given a discrete curve $C$, a set of its consecutive points $C_{i,j}$ where $1 \leq i \leq j \leq |C|$ is said to be a digital straight segment (or $S(i,j)$) iff there exists a digital line $D$ containing all the points of $C_{i,j}$.

$D(a, b, \mu, e)$ is defined as the set of points $(x, y) \in \mathbb{Z}^2$ which satisfy the diophantine inequality:

$$\mu \leq ax - by < \mu + e,$$
Maximal Segments

Definition

Any subset $C_{i,j}$ of $C$ is called a maximal segment iff $S(i,j)$ and $\neg S(i,j+1)$ and $\neg S(i-1,j)$. 
The $\lambda$-MST Estimator (Lachaud et al., 2007)

The $\lambda$-MST, was originally designed for estimating tangents on 2D digital contours. It is a simple parameter-free method based on maximal straight segments recognition along digital contour.

- linear computation complexity
- accurate results
- multigrid convergence
Property

In 3D case, $S(i, j)$ is verified iff two of the three projections of $C_{i,j}$ on the basic planes $O_{XY}$, $O_{XZ}$ and $O_{YZ}$ are 2D digital straight segments.
**Property**

For any discrete curve $C$, there is a unique set $\mathcal{M}$ of its maximal segments, called the tangential cover.
Definition

The set of all maximal segments going through a point \( x \in C \) is called the pencil of maximal segments around \( x \) and defined by

\[
P(x) = \{ M_i \in M \mid x \in M_i \}
\]
The eccentricity $e_i(x)$ of a point $x$ with respect to a maximal segment $M_i$ is its relative position between the extremities of $M_i$ such that

$$e_i(x) = \begin{cases} \frac{\|x-m_i\|_1}{L_i} & \text{if } M_i \in P(x), \\ 0 & \text{otherwise.} \end{cases}$$
The 3D $\lambda$-MST

Definition

The 3D $\lambda$-MST direction $t(x)$ at point $x$ of a curve $C$ is defined as a weighted combination of the vectors $t_i$ of the covering maximal segments $M_i$ such that

$$t(x) = \frac{\sum_{M_i \in P(x)} \lambda(e_i(x)) \frac{t_i}{|t_i|}}{\sum_{M_i \in P(x)} \lambda(e_i(x))}.$$
The function $\lambda$ maps from $[0, 1]$ to $\mathbb{R}_+$ with $\lambda(0) = \lambda(1) = 0$ and $\lambda > 0$ elsewhere and need to satisfy convexity/concavity property.

\[
\sin(\pi x) \quad 64 \left(-x^6 + 3x^5 - 3x^4 + x^3\right) \quad \frac{2}{e^{15(x-0.5)} + e^{-15(x-0.5)}}
\]
Treofil Knot < \cos(2t)*(3+\cos(3t)), \sin(2t)*(3+\cos(3t)), \sin(3t) >
Multigrid Convergence - Trefoil Knot

Lambda: $64(-x^6 + 3x^5 - 3x^4 + x^3)$
Lambda: $2/(\exp(15(x-0.5)+\exp(-15(x-0.5))))$
Lambda: $\sin(3.14x)$
Multigrid Convergence - Treofil Knot

Lambda: 64(-x^6 + 3x^5 - 3x^4 + x^3)
Lambda: 2/(exp(15(x-0.5)+exp(-15(x-0.5))))
Lambda: sin(3.14x)
Introduction

Discrete Straight Segments

λ-Maximal Segment Tangent Direction

Experimental Validation

Tangent direction, x axis, resolution 70 - Treofil

Theoretical Tangent
Lambda-MSTD

Tangent direction, y axis, resolution 70 - Treofil

Theoretical Tangent
Lambda-MSTD

Tangent direction, z axis, resolution 70 - Treofil

Theoretical Tangent
Lambda-MSTD

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**Introduction**

**Discrete Straight Segments**

\[ \lambda - \text{Maximal Segment Tangent Direction} \]

**Experimental Validation**

Viviani < \( \cos(t), \sin(t), \cos(t)^2 \)>

Helix < \( \sin(t), \cos(t), t \)>

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**Convergence Speed - Viviani**

\[ \text{Lambda-MSTD} = 0.4 \times x^{(-0.65)} \]

**Convergence Speed - Helix**

\[ \text{Lambda-MSTD} = 0.5 \times x^{(-0.73)} \]
We have proposed a new tangent estimator for 3D digital curves which is an extension of the 2D $\lambda$-MST estimator.

- We keep the same time complexity and accuracy as the original algorithm
- Asymptotic behavior evaluated experimentally on several space parametric curves is promising
Thank You for your attention!

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