Teaching proofs for freshmen with Coqweb

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PATE 07

First-rate proof assistants

allow highly respectable proofs

- FTA
- Four color theorem
- Jordan curve theorem
- Lambda-Calculus = initial exponential monad

but require

- a logic-oriented way of thinking
- days (weeks?) of training

hence are not (yet?) convenient for freshmen

.

Coqweb

is an free-software interface

- designed for teaching
- Coq-based
- web-based
- developed in ocaml and php
- by Loïc Pottier and Co.

Main features of Coqweb

- a language of statements
  fairly close to the standard mathematical language
  (thus full of ambiguities)
- a language of proof (tactics)
  fairly close to the standard mathematical practice
- proofs are (essentially) performed by clicking
- it can be coupled with Wims, hence
- teachers can include Coqweb exercises
  into their Wims class sheets.

Coupling Coqweb with Wims

has been experimented

- for three years
- at the university of Nice-Sophia-Antipolis
- with the regular freshmen
  (in mathematics and computer sciences)
- via a small amount of proving activity.

WikiCoqweb

is an additional wiki where

- (expert) teachers may enter
  - mathematical statements in a declarative style
  - exercises
  - using Latex
- students may
  - read courses
  - try exercises (without feedback)

The design of Coqweb

aims at

- being accepted by typical students
- being accepted and used by typical teachers who may incidentally add new exercises
- being extended by expert teachers who may add new theories.

Accordingly Coqweb offers two modes

- the student mode, where students (or teachers)
  - read theories
  - solve exercises
- the teacher mode, where teachers
  - write theories
  - add exercises.

with focus on the student mode.

Plan of the talk

1. firstly, we review the position of teachers, students and logicians with respect to proofs
2. secondly, we then discuss what can be a good notion of proof for students
3. thirdly, we stress some strong points of Coqweb

and finish with a small demo.

Teachers in maths

many of them at least

- do not know logic
- do not understand set theory
- are satisfied with their very vague notion of proof
- teach proofs by the method of Leo Lacroix

because

- they were not taught (logic and proofs)
- the method of Leo Lacroix was successful with them
- no other method is available
- huge syllabi leave no room for teaching proofs

Students in maths

too many of them at least

- do not learn rigor
- quit without degree
  (not only because of proofs: see objectif70)

while

”In order to learn a complex process such as proof and disproof, effective integration of new modes of thought with pre-existing contradictory modes is a major undertaking. It is not surprising that easy solutions have not yet been discovered” (Susanna Epp 2003).

Cocorico

Claim:

A precise notion of proof is a crucial ingredient when trying to teach proofs for freshmen.

Question:

what are doing logicians in order to fill the gap between the abstract notions of proof and the daily activity of mathematicians?

Answer:

this task is essentially handled by the theorem-prover community, in particular the Coq team.

Interlude

1. e4 c5 2. N f3 d6 3. B b5+ B d7 4. B xd7+ Q xd7 5. c4 N c6 6. N c3 N f6 7. O -O g6 8. d4 cxd4 9. N xd4 B g7 10. N 4e2 Q e6 11. N d5 Q xe4 12. N c7+ K d7 13. N xa8 Q xc4 14. N b6+ a xb6 15. N c3 R a8 16. a4 N e4 17. N xe4 Q xe4 18. Q b3 f5 19. B g5 Q b4 20. Q f7 B e5 21. h3 R xa4 22. R xa4 Q xa4 23. Q xh7 B xb2 24. Q xg6 Q e4 25. Q f7 B d4 26. Q b3 f4 27. Q f7 B e5 28. h4 b5 29. h5 Q c4 30. Q f5+ Q e6 31. Q xe6 K xe6 32. g3 f xg3 33. f xg3 b4 34. B f4 B d4+ 35. K h1 b3 36. g4 K d5 37. g5 e6 38. h6 N e7 39. R d1 e5 40. B e3 K c4 41. B xd4 e xd4 42. K g2 b2 43. K f3 K c3 44. h7 N g6 45. K e4 K c2 46. R h1 d3 47. K f5 b1= Q 48. R xb1 K xb1 49. K xg6 d2 50. h8= Q d1= Q 51. Q h7 b5 52. K f6+ K b2 53. Q h2+ K a1 54. Q f4 b4 55. Q xb4 Q f3+ 56. K g7 d5 57. Q d4+ K b1 58. g6 Q e4 ?? 59. Q g1+ K b2 60. Q f2+ K c1 61. K f6 d4 62. g7

What is a game of chess

- A game of chess consists of a legal sequence of moves, yielding a legal sequence of positions.
- The sequence of positions enlightens the corresponding sequence of moves.
- Moves are simple, positions are complex.
- Understanding a game of chess is much more than checking that all moves were legal.

What would you think of a teacher teaching chess

without any chessboard?

Claims

- A proof is a sequence of positions and/or moves (called tactics) in a subtle game.
- The sequence of positions enlightens the corresponding sequence of moves.
- Moves and positions are extremely complex (for a freshman).
- Understanding a proof is much more than checking that all moves were legal.

When teaching proofs,we have to

- say what is a proof
- show positions.

Procedural versus declarative proofs

A formal proof may be:

- a sequence of deductions (Hilbert)
- a lambda-term (Curry-Howard)
- a script of tactics (procedural vision, e.g. Coq)
- a script of declarations (in Mizar, see also [AS][C][S]).

Procedural or declarative or even both (F. Wiedijk)?

This is a serious debate among serious people in the theorem-prover community:
to choose the right amount of information.

Claim

For students, an informal notion of proof mentioning altogether states (lists of goals) and transitions (performed by tactics) is adequate.

Interlude: How to show positions?

On the blackboard?

Too difficult!

On the computer-screen?

Too easy!

Concepts versus images

Drawback of the method of Leo Lacroix:

the lack of helpful concepts and associated images.

Coqweb's student window offers persistent areas (hence images) for the following key concepts related to proofs:

- states
- purposes
- contexts
- goals
- variables
- hypotheses
- tactics

Formal versus informal proofs

Typical teachers can only be interested in a tool for teaching informal proofs.

For them, teaching formal proofs is relevant only if it is a way to teach informal proofs.

To this effect, Coqweb offers the guided mode, where

- the student is offered a hint all along the proof
- the hint may take the form of an informal proof so that the student has to understand it in order to translate it into a formal one.
- or the hint may be totally explicit in terms of tactics, in which case the student may be asked to replay the proof (without hint) or to write down an informal proof.

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[a] For what happened then, Google LeBlogDuB