A Controlled Language

for Software Specifications & Mathematical text

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Outline

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Problem 1

- Specifications & standards written in natural languages (RFCs, ISO, ANSI, patents etc)
- Often incomplete or imprecise[1]
- Considerable room for errors and misunderstandings

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- Considerable room for errors and misunderstandings
- Formal methods(formal specs) not readily accepted
- Formalisation is hard
  - Hard to understand & difficult to relate
  - Learning time - model checkers, theorem provers

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Both problems are quite similar

Can we solve these similar problems together?

Math & Specifications: Together?

Sentence level: both have similar structure

Natural language + formal expressions & notations

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Sentence level: both have similar structure

Natural language + formal expressions & notations

- If s is a scheduler state and ... then the scheduling algorithm S is non starving.
- If y is even then $2y$ is also even.
- q_0 and q_1 are empty.
- x and y are positive.
- For all s, if s is a valid scheduler state then ...
- For all x, if x is an even integer then ...
- There exists p such that the running process of ss_p is i.
- There exist two distinct projections of a set S.
- $ss_0 = s$ and $ss_{n+1} = S(ts(n), ss_n)$
- $F_n = F_{n-1} + F_{n-2}$.

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  - Imperative programming
    e.g. $x = x + y$ means $x(t+1) = x(t) + y(t)$
  - Differential equations
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- A generic technology built for math could be extended for specs
  - With some exceptions
    e.g. records in computer science vs. complex numbers in mathematics

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  - References e.g. the last statement
  - Collective vs. distributive readings e.g. positive vs. equal

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  - References e.g. the last statement
  - Collective vs. distributive readings e.g. positive vs. equal
- Automatic translation into a formal language
  such as first-order logic
- Human acceptable & machine understandable
- Combines natural language with formal methods
- Easier to read than a formal language

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But even it fails:

- Could be the first step to implement full NL grammar
- Problems: complexity issues

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In this talk

- A Proof from elementary number theory – working prototype
- Formal Specifications of a simple scheduler – in progress
- Mostly linguistic issues

Outline

1. Motivation
2. Examples
3. Implementation details

An Appetiser (1), In progress . . .

Definition.

A scheduling algorithm S is non starving if

1. s is a scheduler state.
2. i is a process id in some queue of s.
3. For any ts, a sequence of termination state, we define the sequence ss by
    1. $ss_0 = s$
    2. $ss_{n+1} = S(ts(n), ss_n)$

Then there exists p such that the $running\_process$ of ss_p is i.

Supporting Definitions

Definition 1. A process id is a natural number.
Definition 2. A queue is a list of process ids.
Definition 3. A scheduler state is a tuple
$(queues, counter, running\_process, running\_queue\_no)$ where

1. *queues* is list of a queue of size 3.
2. *counter* is a natural number less than 6.
3. *running_process* is a process id.
4. *running_queue_no* is a natural number less than 3.

Definition 4. A scheduler state is correct if the head of its *queues* numbered *running_queue_no* is equal to its *running_process*.
Definition 5. A termination state is an integer which is either TERMINATED, PREEMPTED or YIELD.
Definition 6. A scheduler state is empty if all the queues of its *queues* are empty.

Definition 7. A scheduling algorithm is a function taking a correct scheduler state and a termination state, returning a correct or empty scheduler state.

An Appetiser (2)

Theorem.
If x and y are two even integers then $x + y$ is even.
Proof.
Suppose that x and y are two even integers.
By the definition of even numbers, $x + y = 2a + 2b$ holds.
We deduce that $x + y = 2(a + b)$ by the last statement.
Thus $x + y$ is an even integer because it is a multiple of 2.

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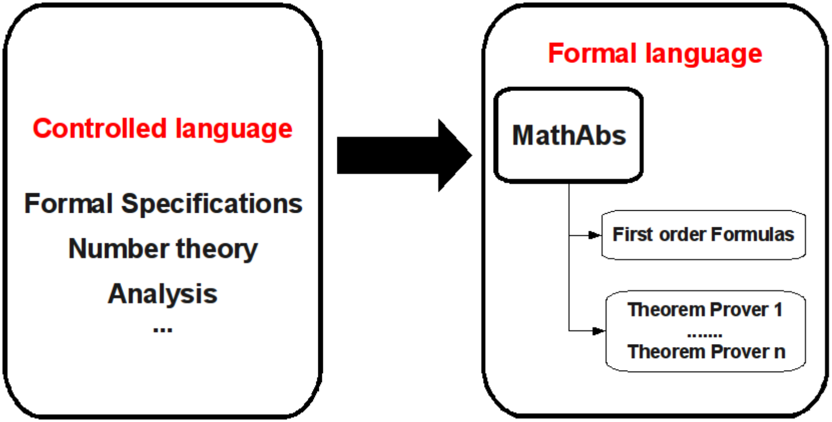
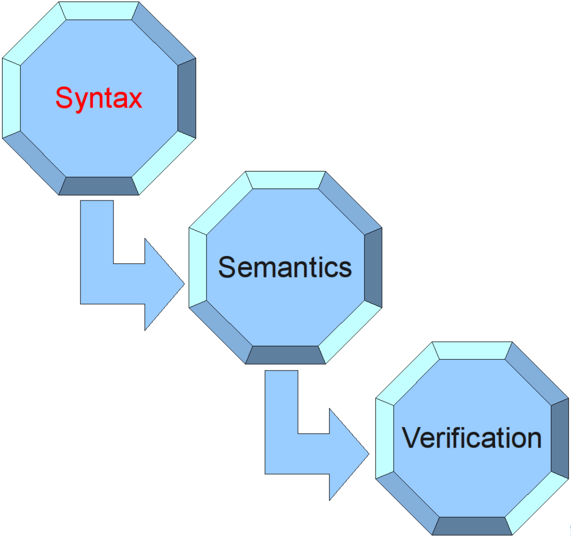
Overall picture of the project



Automatic formalisation in three steps



1. Syntax

Sentence level grammar (without context)

- Natural language + formal expressions & notations
- Type theoretic controlled grammar in Grammatical Framework
- Programming language to define NL grammars
- Our grammar: mostly BNF + dependant records

- ...
- We deduce that $x + y = 2(a + b)$ by the last statement.
- Thus $x + y$ is an even integer because it is a multiple of 2.

1. Syntax

Example: x and y are two even integers.

Abstract Syntax
Syntactic structure/tree

cat Prop, Type, Property, Quant
fun
MkProp : Subj → Quant → [Property] → Type → Prop
Two : Quant
Even : Property
Integer : Type

- ...
- We deduce that $x + y = 2(a + b)$ by the last statement.
- Thus $x + y$ is an even integer because it is a multiple of 2.

Concrete Syntax Linearization rules to each function

Number = Sg | Pl
Type = {s : Number => Str}
Integer = table {Sg => "integer" ; Pl => "integers" }

Property, Quant, Prop = {s : Str}
Two = "two"
Even = "even"

MkProp subj quant props type = subj.s ++ be.s!subj.n ++ quant.s ++ props.s ++ type.s!subj.n

1. Syntax

Modular structure of the grammar allows to rephrase sentences

- We deduce that $x + y = 2(a + b)$ by the last statement.
- By the last statement, we deduce that $x + y = 2(a + b)$.
- By the last statement, $x + y = 2(a + b)$ holds.
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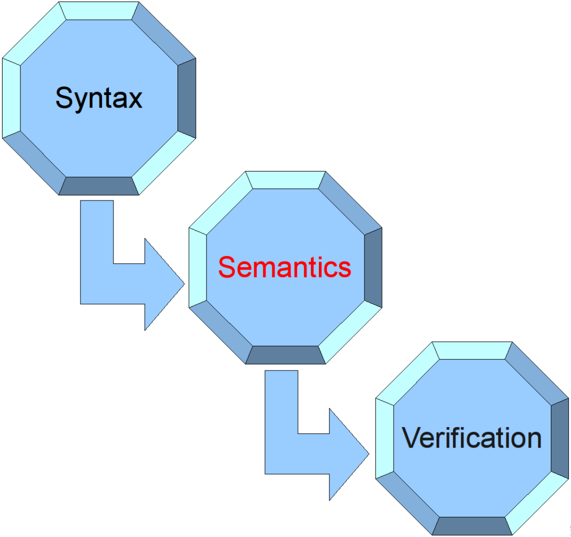

2. Semantics

- Building discourse

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Proof. Suppose that x and y are two even integers. By the definition of even numbers, $x+y=2a+2b$ holds. We deduce that $x+y=2(a+b)$ by the last statement. Thus $x+y$ is an even integer because it is a multiple of 2.

Variables	Type	Gender	Number	How Decl
2	(NoType	Neut	Sg	DeclMan)
it	(?	?	?	?)
$x+y$	(Int	Neut	Sg	DeclMan)
x, y	(Int	Neut	Pl	DeclMan)
a, b	(NoType	Neut	Pl	DeclAuto)
x, y	(Int	Neut	Pl	DeclMan)
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- Solving basic anaphora e.g. It and They

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Formulas	Type
$x + y \in Z$ & even $(x + y)$	Deduction
$x + y = 2(a + b)$	Deduction
$x + y = 2a + 2b$	Deduction
$x, y \in Z$ & (even (x) & even (y))	Hypothesis
$\forall_{x,y}(x, y \in Z$ & even (x) & even $(y) \Rightarrow$ even $(x + y))$	Goal

2. Semantics

- Distributive vs. Collective readings
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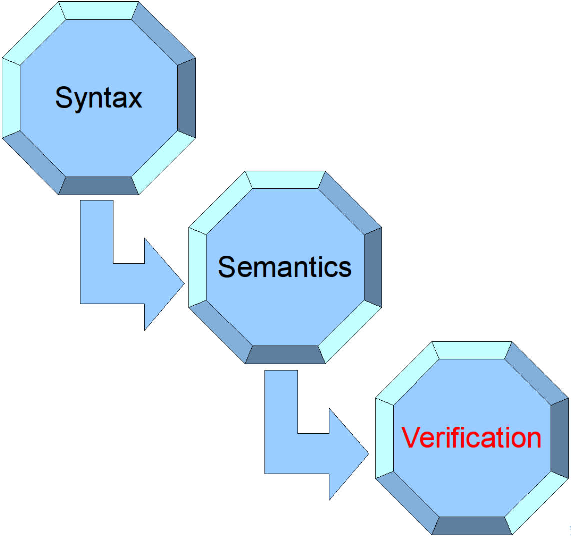
- Translation of controlled language to an abstact mathematical langauge MathAbs

Automatic formalisation in three steps



3. Verification

- MathAbs to first order formulas in progress . . .

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- MathAbs to first order formulas in progress ...
- MathAbs to a prover specific formalism after PhD
  - Logical types and linguistic types are not the same
  - Need a theorem prover that could deals with types as predicates
  - e.g. PML[a]

[a] http://www.lama.univ-savoie.fr/tracpml

Summary

- Specifications and Math text are similar problems
- A controlled language could work for both
- A long term working project, but feasible

Questions

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