Classification of p-adic compact manifolds

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Let p be a prime number. We consider the p-adic absolute value over $\mathbb{Q}$:

$$\begin{array}{rcccc} |.|_p & : & \mathbb{Q} & \to & \mathbb{R} \\ & & x & \mapsto & p^{-v_p(x)} \end{array}$$

where $v_p(x)$ is the p-adic valuation of x. The field of *p-adic numbers*, denoted by $\mathbb{Q}_p$, is by definition the completion of $(\mathbb{Q}, |.|_p)$. The distance induced by this absolute value is ultrametric, giving to $\mathbb{Q}_p$ very different topological properties then those of the field of real numbers $\mathbb{R}$.

The first objective of this master thesis will be to study the topological properties of $\mathbb{Q}_p$, the notion of p-adic integrals and the notion of p-adic analytic manifolds, see for instance [1], [2] et [4].

Then, we will consider a theorem by Serre classifying the compact p-adic manifolds :

Theorem. *(Serre [3]).*

1. *Any compact p-adic manifold is isomorphic to a disjoint union of r p-adic balls of radius 1 and dimension n, denoted by $r.B^{(n)}$, for a convenient integer $r \geq 1$.*
2. *The manifolds $r.B^{(n)}$ and $r'.B^{(n)}$ are isomorphic if and only if $r = r' \mod p-1$.*

Then, if we attach to X the class of r modulo $p-1$, we obtain an element $i(X)$ of $\mathbb{Z}/(p-1)\mathbb{Z}$, called *$p$-adic Serre invariant*, which is an invariant of X and characterises X up to isomorphism.

At the end, we could consider these questions in the setting of complex formal series $\mathbb{C}[[t]]$, using the theory of *motivic integration* introduced by Kontsevich in the 90's. Unlike $\mathbb{Q}_p$, the ring $\mathbb{C}[[t]]$ which is the t-adic completion of $\mathbb{C}[t]$, is not locally compact and does not have an Haar measure. The motivic integration is then an analogous of the p-adic integration but the values of the integral are not real but *motives*.

Références

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[4] Jean-Pierre Serre. *Lie algebras and Lie groups*, Lecture Notes in Mathematics, 1500, 1964 lectures given at Harvard University, Corrected fifth printing of the second (1992) edition, Springer-Verlag, Berlin, 2006.