Around the Milnor fiber

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Let f be a complex polynomial in n variables. Assume that f has an isolated critical point at x_0 and denote by a the value $f(x_0)$.
Milnor [4] shows the existence of two reals ε and η with $1 >> \varepsilon >> \eta > 0$ such that

$$f \quad : \quad f^{-1}(B(a,\eta) \setminus \{a\}) \cap B(x_0,\varepsilon) \subset \mathbb{C}^n \quad \to \quad B(a,\eta) \setminus \{a\} \subset \mathbb{C}$$

is a C^∞ locally trivial fibration.
The fiber of this fibration is called *Milnor fiber of* f *at* x_0 denoted by F_{x_0}.

This fiber has the homotopy type of a bouquet of μ spheres of dimension $n-1$. This number μ is called *Milnor number of* f at x_0 and Milnor proves the equality

$$\mu = \dim \frac{\mathbb{C}\{x_1, ..., x_n\}}{(\frac{\partial f}{\partial x_1}, ..., \frac{\partial f}{\partial x_n})}.$$

If f is a convenient and non degenerate polynomial for its Newton polyhedron Γ_{x_0}, Kouchnirenko [3] computes this number in terms of volumes associated to Γ_{x_0}. Finally, considering a resolution of singularities of f, A'Campo [1] express this Milnor number in terms of the combinatory of the exceptional divisor in the resolution.

The objective of this master thesis will be to study the proofs of above theorems and could be continue by the consideration of other invariants attached to the singularity x_0 for instance those induced by the *monodromy action* of the fundamental group $\pi_1(B(a,\eta) \setminus \{a\})$ on the Milnor fiber F_{x_0}. Finally we could consider the *motivic Milnor fiber* S_{f,x_0} of Denef–Loeser [2], which is a *motive* containing several invariants of the Milnor fiber F_{x_0} and constructed using formal arcs with origin in x_0.

Références

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